

# EEG MULTIREOLUTION ANALYSIS USING WAVELET TRANSFORM

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**Abstract:** Wavelet transform (WT) is a new multiresolution time-frequency analysis method. WT possesses well localization feature both in time and frequency domains. It acts as a group of band-pass filters to decompose mixed signal into signals at different frequency bands. EEG, as a noninvasive testing method, plays a key role in the diagnosing diseases, and is useful for both physiological research and medical applications. Using the dyadic wavelet transform, the EEG signals are successfully decomposed to the alpha rhythm (8-13Hz), beta rhythm (14-30Hz), theta rhythm (4-7Hz), and delta rhythm (0.3-3Hz), and the EMG trembles in EEG are effectively removed while the useful information of EEG are well reserved so as to improve SNR. The experiment results are given in the end of the paper.

**Key words:** Electroencephalograph (EEG), Time-Frequency Analysis (TFA), Wavelet Transform (WT), Decomposition, Denoising

## 1. INTRODUCTION

Electroencephalogram (EEG) is the summed electrical activity of very large numbers of neurons. It can be recorded from scalp electrodes in human subjects or from electrodes implanted in specific brain regions of experimental animals. EEG, as a noninvasive testing method, plays a key role in the diagnosing diseases, and is useful for both physiological research and medical applications. A lot of signal processing techniques have been widely applied to the analysis of clinical EEG signal. As a conventional method, Fourier transform has been widely used for the spectral analysis. The validity of the Fourier technique depends on the hypothesis that the EEG is stationary. In practice, EEG

signal is generally time-varying, nonstationary, sometimes transient, and usually corrupted by noise. Hence it should be usually treated as random or stochastic processes.

Recent years, the time-frequency analysis (TFA) has been successfully applied in some biomedical signals to detect both temporal and spectral features of biomedical signals. Wavelet transform (WT) is one of the TFA, and has been used successfully in many applications. In this paper, wavelet transform is applied to analyze and decompose the time-varying and nonstationary EEG signal, investigate its time-frequency characteristics, and remove the noise.

## 2. METHODS

Wavelet transform is a new multiresolution time-frequency analysis method. WT possesses well localization feature both in time and frequency domains. It acts as a group of band-pass filters to decompose mixed signal into signals at different frequency bands.

The complex valued function  $\Psi(x)$  is said to be a wavelet if its Fourier transform  $\Psi(\omega)$  satisfies:

$$\int_0^{+\infty} \frac{|\Psi(\omega)|^2}{\omega} d\omega = c_\Psi < +\infty$$

Let  $\Psi_s(x) = (1/s) \Psi(x/s)$  be the dilation of  $\Psi(x)$  by the scale factor  $s$ . The wavelet transform of a function  $f(x)$  of  $L^2(\mathbb{R})$  is defined by

$$Wf(s, x) = f(x) * \Psi_s(x)$$

Mallat fast algorithm of wavelet transform is used to decompose and reconstruct EEG signals. Adopting this algorithm, signals are decomposed to

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wavelet coefficients in different scales. Some coefficients related to noises are discarded, and the remained signals are reconstructed by inverse WT, then noises can be removed.

Mallat proved that the average density of modulus maxima of a white noise is inversely proportional to the scale  $s$  of WT. If the Lipschitz exponent of  $f(x)$  is  $\alpha$ , there exists a constant  $K$  and

$$|Wf(s, x)| \leq Ks^{-\alpha}$$

If let the scale  $s=2^j$ , then we have

$$|Wf(2^j, x)| \leq K2^{j\alpha}$$

Since EEG has singularities with positive Lipschitz exponent and the Lipschitz exponent of noise is negative, WT of EEG and noises present different

inclination with the increase of scale. Energy of noises concentrate on  $2^1$  scale and decrease significantly when the scale increases, while EEG concentrates mainly on scales  $2^2$ - $2^5$ . By eliminating wavelet coefficients of small scales, denoised EEG are reconstructed by other scales, and the useful signals of EEG are well reserved while noises are removed effectively.

### 3. RESULTS

Using the dyadic wavelet transform, we successfully decompose the EEG signals to the alpha rhythm (8-13Hz), beta rhythm (14-30Hz), theta rhythm (4-7Hz), and delta rhythm (0.3-3Hz) as shown in Fig.1, and effectively remove the EMG trembles in EEG while the useful information of EEG are well reserved so as to improve SNR as shown in Fig. 2.

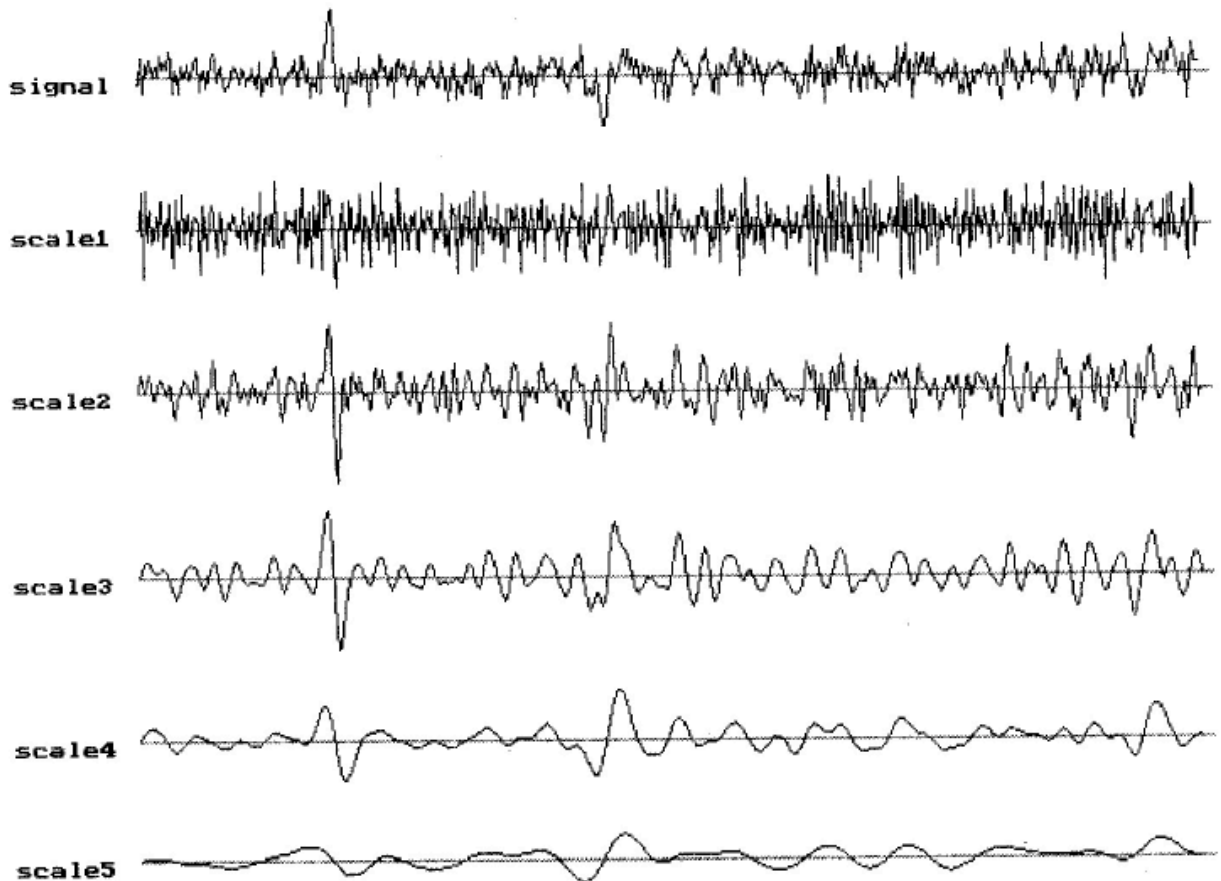


Fig.1 Noisy EEG signal and its wavelet transform at different scales.  
Scale1 to scale5: dyadic wavelet transform of EEG signal from scale  $2^1$  to  $2^5$ .

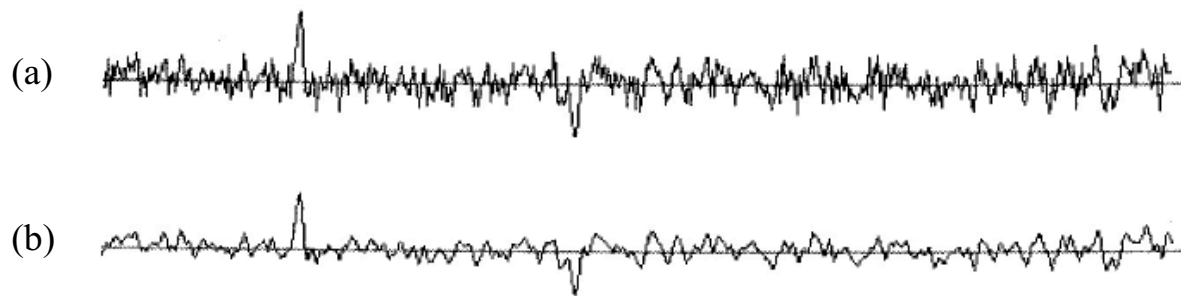


Fig.2 EEG and its denoising: (a) EEG signal with noise; (b) denoised EEG.

#### 4. CONCLUSION

The main advantage of wavelet transform is to provide simultaneous information on frequency and time location of the signal. EEG, as a noninvasive testing method, plays a key role in the diagnosing diseases, and is useful for both physiological research and medical applications. By wavelet transform, EEG can be decomposed into different detail components or various frequency bands, and the noise also can be rejected effectively according to the different behavior of WT coefficients of signal and noise. Doctors may use those detail components and denoised EEG for further clinical analysis. It is indicated that WT provides a promising method to characterize the EEG and remove the noise.

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